hw set 15
sect 6.3
p465

1. $y = 2-3x \,,\; -2 \le x \le 1$

a) use Arc Length Formula

$$L = \int_{-2}^{1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{1} \sqrt{1+(-3)^2}\, dx$$

$$= \sqrt{10}\,(1-(-2)) = 3\sqrt{10}$$

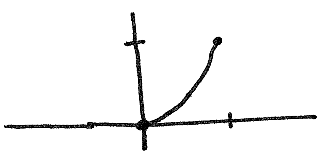
b) use distance formula:

$(-2,8)$ to $(1,-1)$

$$d = \sqrt{(-2-1)^2+(8+1)^2}$$

$$= \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

7. $x = y^{3/2} \,,\; 0 \le y \le 1$



$$L = \int_0^1 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^1 \sqrt{1+\left(\tfrac{3}{2}y^{1/2}\right)^2}\, dy$$

$$= \int_0^1 \sqrt{1+\tfrac{9}{4}y}\, dy$$

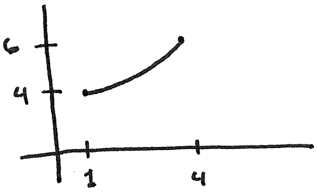
$$= \frac{1}{2}\int_0^1 \sqrt{4+9y}\, dy \qquad u = 4+9y \quad du = 9\,dy$$

$$= \frac{1}{18}\int \sqrt{u}\, du = \frac{1}{18}\cdot\frac{2}{3}\cdot u^{3/2}$$

$$= \frac{1}{27}u^{3/2} = \frac{1}{27}(4+9y)^{3/2}\Big|_0^1$$

$$= \frac{1}{27}\left[13^{3/2}-8\right] = \frac{13\sqrt{13}-8}{27}$$

5. $\begin{cases} x = 1+3t^2 \\ y = 4+2t^3 \end{cases}$ $\quad 0 \le t \le 1$



$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(6t)^2+(6t^2)^2}\, dt = \int_0^1 \sqrt{36t^2+36t^4}\, dt$$

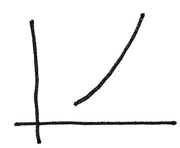
$$= \int_0^1 6t\sqrt{1+t^2}\, dt \qquad u = 1+t^2 \quad du = 2t\,dt$$

$$= 3\int \sqrt{u}\, du = 3\cdot\frac{2}{3}u^{3/2} = 2u^{3/2}$$

$$= 2(1+t^2)^{3/2}\Big|_0^1 = 2\cdot\sqrt{8}-2 = 4\sqrt{2}-2$$

$$= 3.656854$$

8. $y = \frac{x^2}{2} - \frac{\ln x}{4} \,,\; 2 \le x \le 4$



$$L = \int_2^4 \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\, dx$$

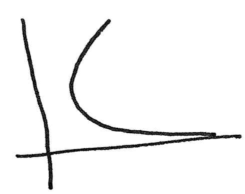
$$= \int_2^4 \sqrt{1+x^2-\frac{2x}{4x}+\frac{1}{16x^2}}\, dx = \int_2^4 \sqrt{\frac{1}{2}+x^2+\frac{1}{16x^2}}\, dx$$

$$= \int_2^4 \sqrt{\frac{8x^2+16x^4+1}{16x^2}}\, dx = \int_2^4 \sqrt{\frac{(4x^2+1)^2}{16x^2}} = \int_2^4 \frac{4x^2+1}{4x}\, dx$$

$$= \int_2^4 x+\frac{1}{4x}\, dx = \frac{x^2}{2}+\frac{\ln x}{4}\Big|_2^4 = \left[8+\frac{\ln 2}{2}\right]-\left[2+\frac{\ln 2}{4}\right]$$

$$= \boxed{6+\tfrac{1}{4}\ln 2}$$

9. $\begin{cases} x = e^t - t \\ y = 4e^{t/2} \end{cases}$ $\quad -8 \le t \le 3$



$$L = \int_{-8}^{3} \sqrt{(e^t-1)^2+(2e^{t/2})^2}\, dt$$

$$= \int_{-8}^{3} \sqrt{e^{2t}-2e^t+1+4e^t}\, dt$$

$$= \int_{-8}^{3} \sqrt{e^{2t}+2e^t+1}\, dt = \int_{-8}^{3} \sqrt{(e^t+1)^2}\, dt$$

$$= \int_{-8}^{3} e^t+1\, dt = e^t+t\Big|_{-8}^{3}$$

$$= (e^3+3)-(e^{-8}-8) = e^3-e^{-8}+11$$